Tutorial 8

ROMA Analysis

This tutorial describes the steps to apply for a multifractal analysis based on rank ordering of fluctuations, method invented by Chang and Wu (2004). Since the Rank Ordered Multifractal Analysis (ROMA) method is rather novel and complex this module includes several sub-modules, some of them with a dedicated technical content aiming to help the user to understand/control some of the particularities of the method itself.

The **ROMA Analysis** module contains 4 sub-modules labeled as follows: a) **Fluctuations**, b) **Range-limited SFs**, c) **ROMA Spectrum** and d) **AROMA**, all of them accessible from the right hand side panel of the main ROMA analysis window illustrated in Figure 1.

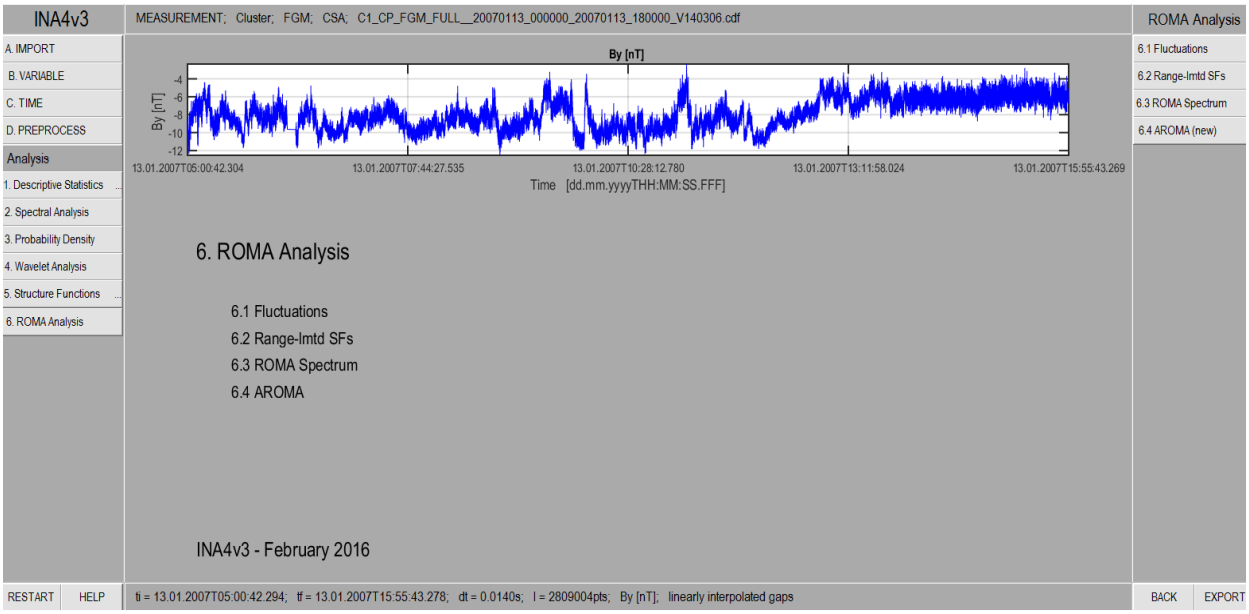

*Figure 1. The main window of ROMA analysis opens when the user clicks the corresponding button in the left panel of INA. The upper plot shows the time series to be analyzed with ROMA. The four buttons at the right are links to the analysis routines devoted to: statistics of the raw and rank ordered fluctuations (**6.1. Fluctuations**); analysis of the range limited structure functions of the rank ordered fluctuations (**6.2. Range-lmtd SFs**); and the analysis of the ROMA spectrum itself (**6.3 ROMA Spectrum** and **6.4 AROMA**).*

ROMA is a complex analysis method, thus the output, the ROMA spectrum, must be supplemented by preliminary tests and analyses to understand and validate the results. Analysis steps like the ones grouped under the labels **Fluctuations** and **Range-limited SFs** are used for such preliminary purposes. The functions **ROMA Spectrum** and **AROMA** on the other hand provide the ROMA spectrum itself using two different approaches.

Even a practical and technical tutorial on ROMA like this one needs an introduction on the specific characteristics of the method itself. ROMA is about a statistical description of fluctuations based on multifractals. If the fluctuations of the analyzed field, x, exhibit monofractal properties, then all PDFs, P (Δx,τ) computed for this field at all scales would collapse onto a single scaling function $P_s(\Delta x)$ and the rescaling procedure is described by one number, s, as illustrated in Tutorial #5 . In practice, however, only parts of the PDFs may collapse on the master curve while the rest remains unscaled. In such cases Chang and Wu (2004) imagined that the scaling factor s may depend on the sizes of scaled fluctuations, i.e. $s = s(Y)$ with $Y = \delta x \cdot (\tau/\tau_0)^{-s(Y)}$, the rescaled, or rank ordered fluctuations, τ_0.

In the original ROMA methodology (implemented in INA by the **ROMA Spectrum** module) one has to find a graphical solution to a transcendental equation:

$$S\left(\Delta x(\tau); Y_1, Y_2\right) = \int_{\Delta x(Y_1)}^{\Delta x(Y_2)} |\, \Delta x(\tau)\,|^q \, P\left(|\, \Delta x\,|, \tau\right) d\Delta x \simeq A\tau^{qs(Y)} \tag{1}$$

and search for the intersection point(s) between ζ_s and $s_{var} \cdot q_{cst}$, where ζ_s is the slope of the structure function S of order q for the value s, computed iteratively for a range of values s between 0 and 1. Note that the structure function in the equation above is computed over a limited range of rank ordered fluctuations, Y. If the fluctuations in the chosen range are not monofractal, or if the scale range τ is not properly chosen, this method may lead to multiple solutions for the same range. The problem of choosing which one of the multiple solutions is the correct one can be solved by checking which one satisfies the best the monofractal scaling law.

In INA we also implemented an alternative minimization approach to compute the ROMA spectrum that at the same time speeds up the computation, and also finds the solution by directly searching for the value *s* for which the monofractal scaling is satisfied for all orders q. In other words the alternative approach searches that value s for which the structure function slopes satisfy $\zeta_q = s_{cst} \cdot q_{var}$ for all q, in a minimization sense. We compiled an automatic algorithm to implement this approach by computing a measure of the difference between ζ_q and $s_{cst} \cdot q_{var}$. This measure, referred to as "normr_ideal", is defined as the root mean square error (RMSE) between the two lines:

$$\text{normr\_ideal} = \sqrt{\sum \left(\zeta_q - s_{cst} \cdot q_{var}\right)^2} \tag{2}$$

, where the sum is over all q. The value of s corresponding to the smallest normr_ideal is the AROMA (Automated ROMA) solution for the analyzed range ΔY.

In the following we describe the functionality of the analysis included in the ROMA module that can be called from the right hand side panel of the main INA window illustrated in Figure 1.

a) Fluctuations

The Fluctuations module has a main window, illustrated by figure 2 below, that includes three functionalities to be called from the panel at the right side of the window: 1) **individual**,

where the user can analyze fluctuations on individual scales, 2) **multiple1**, where we show the maximum amplitude for a range of scales and 10 values of s and 3) **multiple2**, where the user can observe the number of data points within a chosen ΔY range, as a function of scale and s.

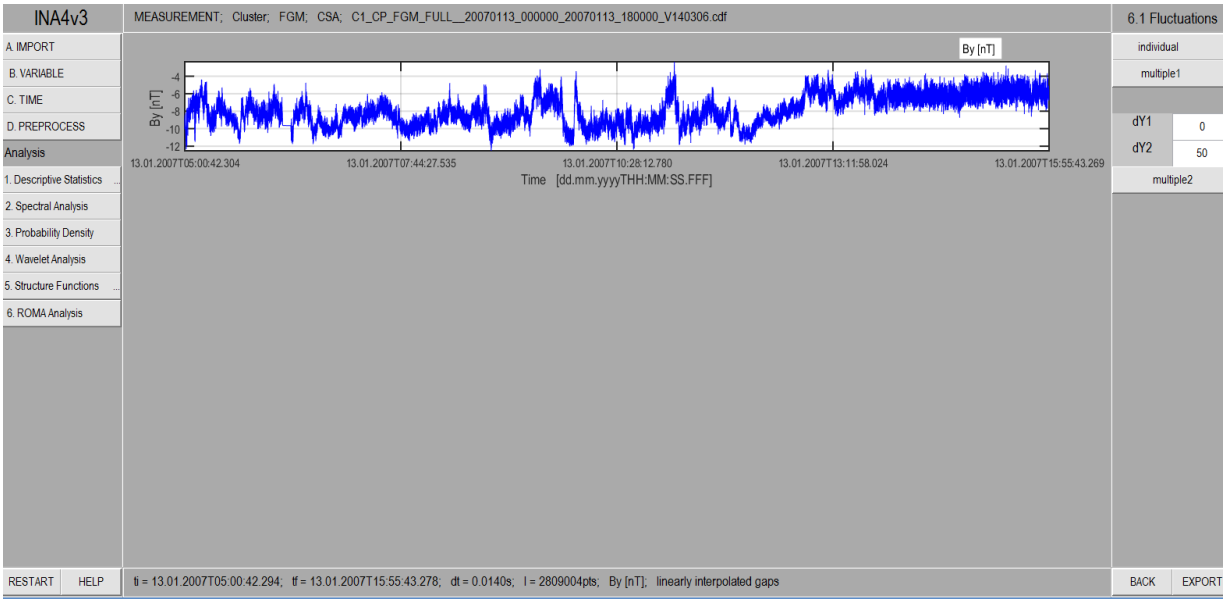

*Figure 2. The window opened for the analysis of fluctuations in the ROMA module. The user can investigate the statistics of the raw and rank ordered fluctuations for one scale chosen by the user (**individual** button), for all the bins (**multiple1**); the user can also analyze an individual bin defined by the user inputs: dY1 and dY2.*

The analysis of the statistics of fluctuations on individual scales provides important insights on the distribution and maximum values of rescaled (rank ordered) fluctuations Y. In the **individual-Fluctuations** sub-module of the **ROMA Analysis** module: **s_roma** is the scaling index used to rescale the fluctuations on scale τ, specified by the parameter **iscale,** with respect to the reference scale τ_0, specified by the parameter **rscale;** the parameters **dY1** and **dY2** define the limits of the analyzed bin/range for the rescaled (rank ordered) variable, ΔY.

Figure 3 is an illustration of the **individual-Fluctuations** window. It shows an example of magnetic field fluctuations from Cluster analyzed on scale **iscale =** 7, rescaled with the reference scale **rscale** = 0 and index **s_roma** = 0.22. The left panel shows the fluctuations (raw, Dy, in blue and rescaled/rank ordered, dY, in red) as amplitude vs number of points. The plot on the right depicts the corresponding PDFs (in continuous lines) for Dy and dY for iscale. The subset of fluctuations corresponding to the range ΔY = [0.3724, 0.5537] is superposed as individual points on the two PDFs.

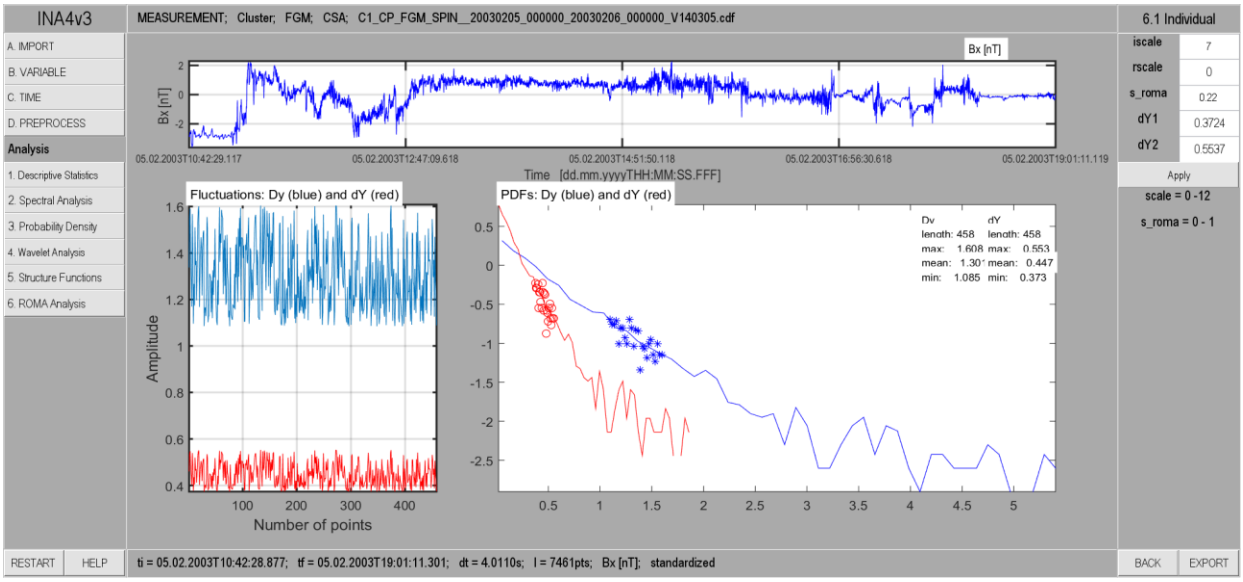

*Figure 3. Illustration of the sub-module –**6.1. Fluctuations** of the ROMA module. The upper plot shows the signal to be analyzed. The left plot shows the histogram of raw (in blue) and rescaled or rank ordered (in red) fluctuations computed with the windowed differences procedure (described in Tutorial #5) for the scale selected by the parameter iscale. The right plot shows the PDF computed for the scale specified by iscale, both for the raw fluctuations (in blue) and rank ordered fluctuations (in red). The PDF values corresponding to a range of Y values specified by the user (through dY1 and dY2) are shown as points overlapped on the PDFs.*

b) **Range-limited SFs**

In addition to the conventional structure function analysis, described in Tutorial #7, the Range limited SFs analysis included in the ROMA Analysis class gives the user the possibility to calculate the structure functions for a limited range of amplitudes determined by the rank ordering of fluctuations. The main window attached to this functionality is shown in Figure 4.

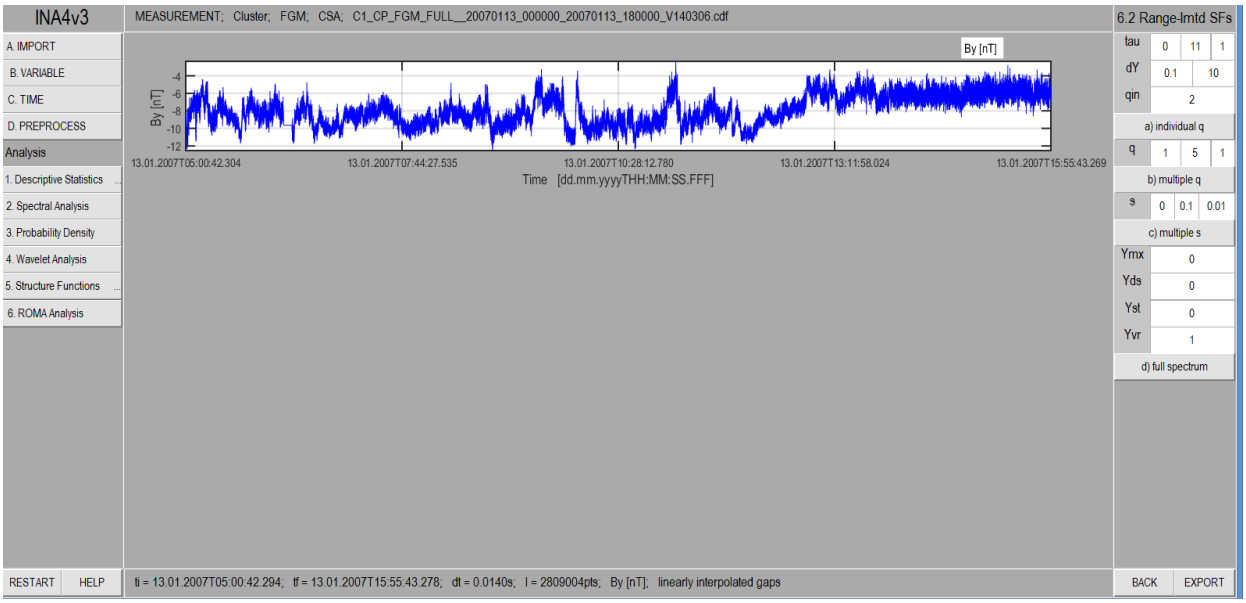

*Figure 4. Illustration of the sub-module –**6.2. Range-lmtd SFs** of the ROMA module. The upper plot shows the signal to be analyzed. The right panel gives access to four sub-modules: **a) individual q** computes the slope of the structure function for a range ΔY and a fixed q specified by the user; **b) multiple q** computes the range limited SF for several orders q, defined by the user, computes also the slope for each q and also makes some tests of the monofractal scaling as discussed in the text below; **c) multiple s** computes the slopes of the range limited SF for a given range ΔY and a range of values s; it shows which value of s satisfies better the monofractal scaling; **d) full spectrum** computes the full ROMA spectrum for all values of Y (based on the AROMA methodology)*

In figure 5 we detail the "**b) multiple q**" functionality of this sub-module. A screenshot from this layer is shown in Figure 5. The left panel of Figure 5 illustrates the range limited structure functions for a given value of s (s=0.22) and a range of q values (between -3 and +3 with increment of 1). The middle panel illustrates the goodness of the monofractal fit using a superimposed plot of ζ_q/q and s as a function of q. The small plot in the top right part depicts the "normr_ideal" parameter, discussed at the beginning of this section. The small plot in the bottom right part depicts the errors of the linear fits of the Structure Functions (left panel), computed as the norm of residuals between structure functions and the corresponding fits; small values for this parameter indicate a good linear approximations for the structure functions.

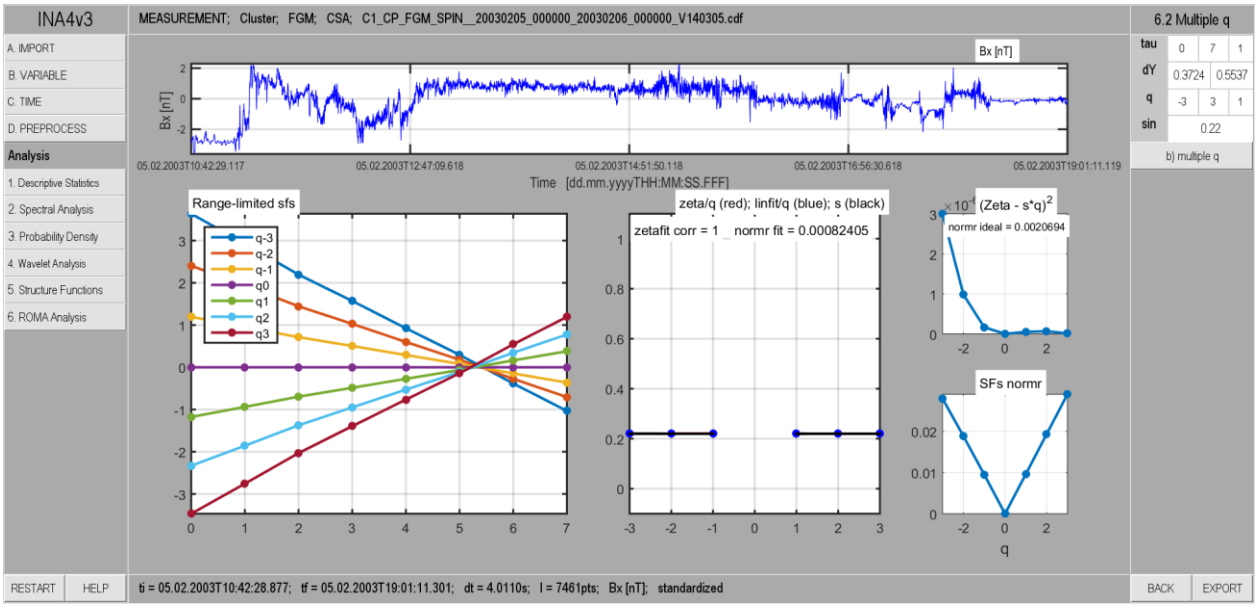

Figure 5. Illustration of the multiple q functionality of the- Range-limited SFs subclass. The left plot shows the range limited structure functions for eight scales computed for the range ΔY=[0.3724, 0.5537] and seven moment orders q; the middle plot shows the ratio ζ_q/s for each q; the upper right plot shows the "normr_ideal" parameter for all q, and the lower right plots shows the goodness of the structure functions linear fits.

c) **ROMA Spectrum**

The ROMA spectrum sub-class contains three functionalities: **1) individual range**, where the user computes the ROMA solution for only one individual range ΔY = [Y1,Y2] where the limits of the interval, Y1 and Y2, are specified by the user, **2) individual q**, where the user computes the classic ROMA spectrum for all Y but only for one value of the order moment q, specified by the user and **3) full spectrum**, where the user computes the full ROMA spectrum for all q. The ROMA spectrum window is illustrated in Figure 6.

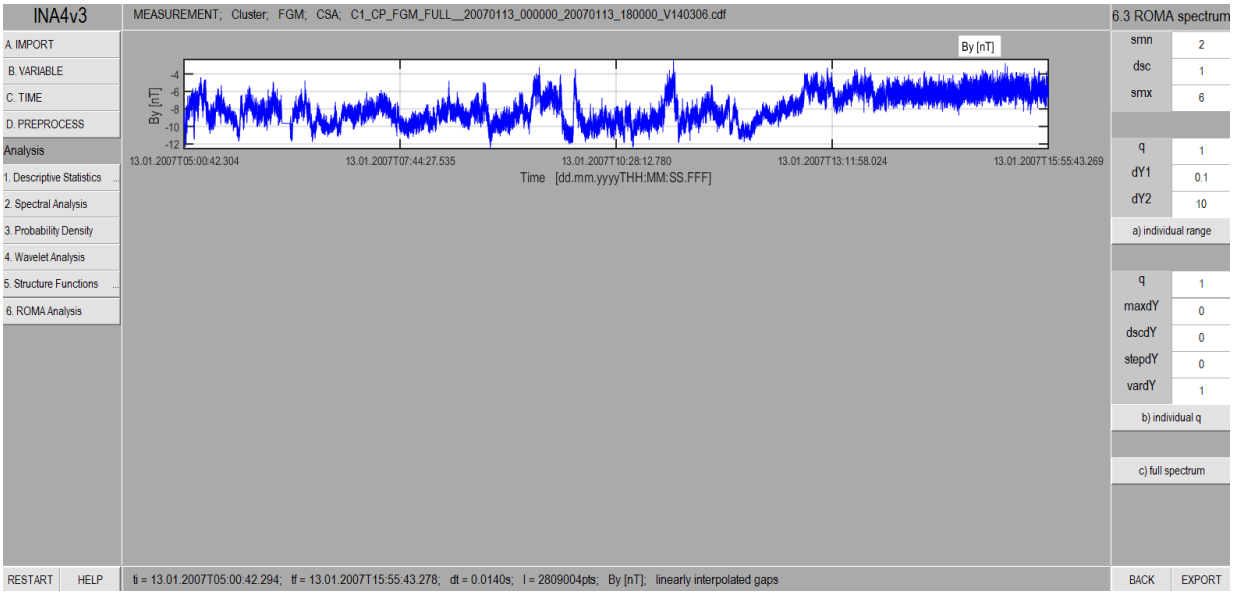

*Figure 6. Illustration of the sub-module – **6.3. ROMA spectrum** of the ROMA module. This sub-module is constructed based on the original procedure described by Chang and Wu (2008). The upper plot shows the signal to be analyzed. The right panel gives access to three sub-modules: **a) individual range** computes the ROMA solution for one bin ΔY=[Y1,Y2], one moment order q and a range of scales, all specified by the user; **b) individual q** computes the ROMA spectrum for all the bins defined for the rescaled variables Y and only one order q, specified by the user; **c) full spectrum** computes the ROMA spectrum for all bins of Y and all q.*

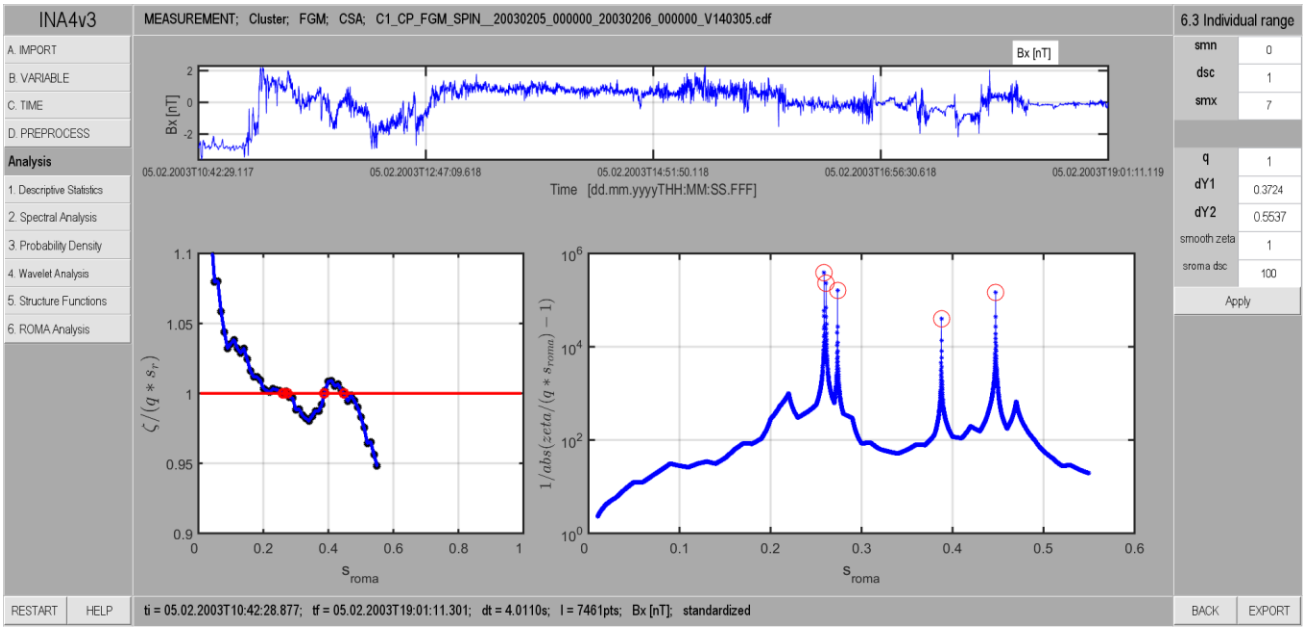

*Figure7. Illustration of the **individual range** functionality of the - ROMA spectrum subclass. The left plot illustrates in blue the values $\zeta_q/(q*s)$ computed for various values of s for the bin ΔY=[0.3724, 0.5637] and q=1. The intersection of the blue curve with the red line identifies the ROMA solution for the respective bin ΔY, i.e. for which values of s the monofractal scaling, $\zeta_q = q*s$, is satisfied. The left plot shows that in the considered bin there are five different solution. The right plot quantifies how close to zero is the quantity $\zeta_q - q*s$.*

A screenshots depicting the **individual range** functionality is shown in Figure 7 for the bin ΔY=[0.3724, 0.5637] and q=1. The plot on the right shows that in the considered bin one would detect five possible solutions, i.e. five different intersection points between $\frac{\zeta_s(s)}{q_{cst}s}$ and $s = 1$. One way to smooth out some of the multiple solutions is to use a smooth function to remove the small scale fluctuations in $\frac{\zeta_s}{q_{cst}}$. The user can test this option by choosing a value for the **smooth_zeta** parameter; usual values for this parameter range between 1 (no smoothing) and 1000 (this value removes most of the small amplitude fluctuations in $\frac{\zeta_s}{q_{cst}}$. Figure 8 illustrates this option for ***smooth_zeta*** *= 2000;* indeed the multiple solutions around ~ 0.27 as well as the solution found around s~ 0.4 turned out to be related to small scale fluctuations and the algorithm suggests only one solution after smoothing, close to s = 0.22.–

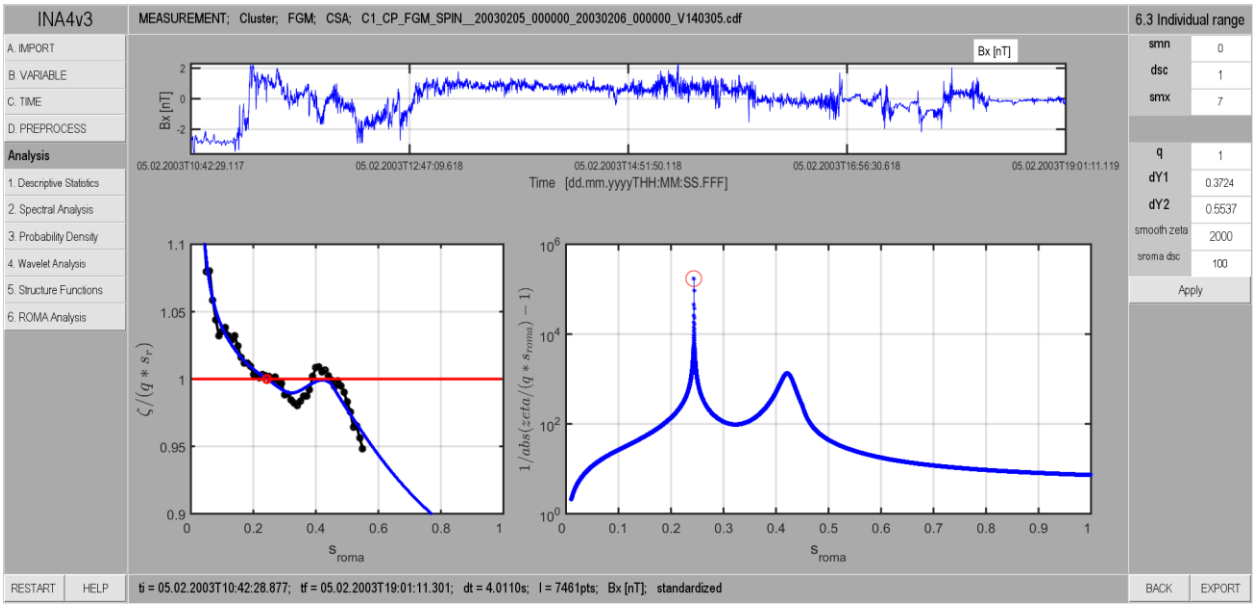

*Figure 8. Illustration of the smoothing procedure applied for the **individual range** functionality of the ROMA spectrum module. The blue curve in the left plot is obtained with the smooth zeta parameter set to 2000 (see the text for details).*

d) **AROMA**

AROMA is a procedure developed in the FP7 project STORM and implements a multi-parametric optimization procedure for ROMA computations aiming to catch the monofractal behavior for each ΔY bin from a global view, over all moment orders q. AROMA includes 3 functionalities: ***lmap***, ***smap*** and ***spectrum***. The first two are developed to help the user to gain some insight on the structure of the data prior to the ROMA analysis itself.

The user can modify the parameters: *tau* (the reference scale, the largest scale and discretization of scales); *q* (the minimum, the maximum and the discretization of the moment orders); *s* (the minimum, maximum and the discretization value of the s parameter, the ROMA index) and the minimum and the maximum value of ΔY.

The parameters identified by "**Y-pr, dr**" describe how the ranges ΔY are defined:

- "*pr*" defines how the ranges of Y are distributed between the two values minY and maxY; the user must use pr = 1 for equally spaced Y bins, or larger values for non-uniform binning; pr = 2 and pr = 3 define a quadratic and respectively cubic growth of the bin sizes.
- "*dr*" controls the number of ranges between [minY, maxY], e.g. dr = 0.2 sets an interval of 5 bins, or dr = 0.1 to set it of 10 bins.

Screenshots from the 3 layers of AROMA are illustrated in Figures 9, 10 and 11.

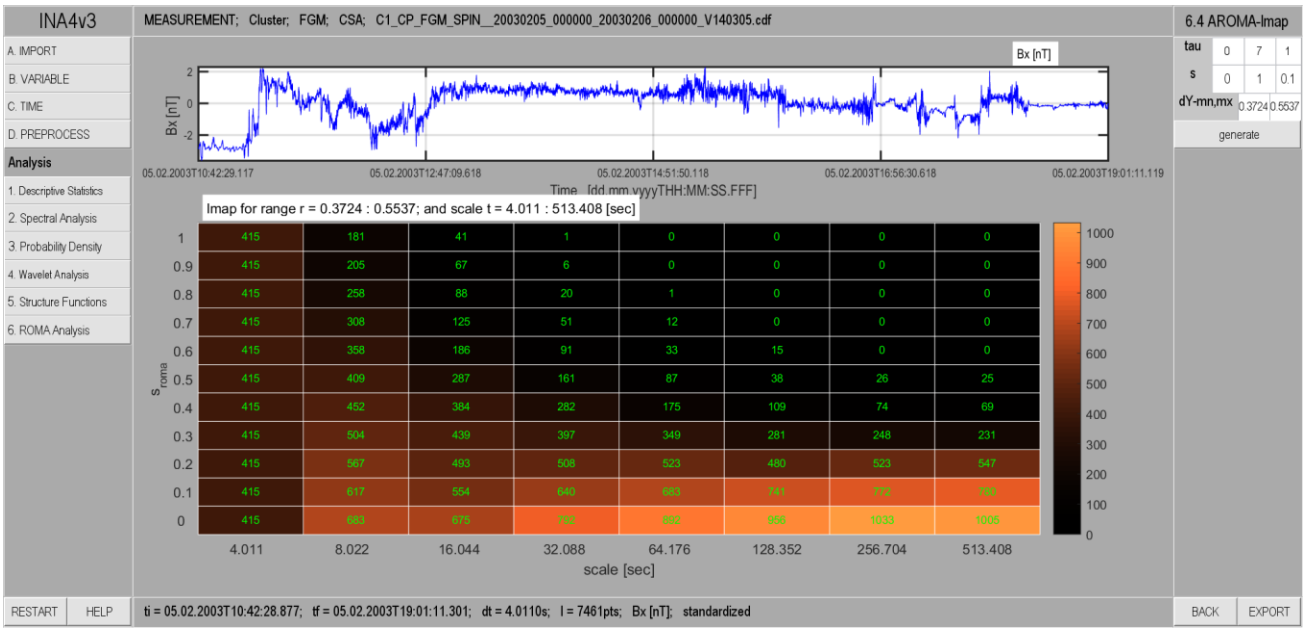

Figure 9. Illustration of the lmap feature of the AROMA submodule. The central diagram illustrates how many points (or fluctuations or differences) are assigned to the bin ΔY=[0.3724, 0.5537]. (see the text for details).

The length maps (***lmap***) window in AROMA shows the number of points inside one given bin ΔY, and how are they distributed as a function of scale *tau* and values of *s*. The figure 9 shows an example for the bin ΔY=[0.3724, 0.5537]. The number of data points inside each bin gives us a direct measure of the accuracy of the AROMA solution for that range ΔY.

The solution maps (***smap***) depicted in Figure 10, compares the "classical" ROMA solutions (left, implemented by the submodule 6.3) with the AROMA methodology (right) described in the beginning of this tutorial and based on the minimization of the "normr_ideal" function. On the left, the brightest colors indicate the "peaks" of the classical solutions (corresponding to the peaks of the right hand side plot of Fig. 7 or 8), with s on the y axis and q on the x axis. One notices a rather broad band of bright colors around s ~ 0.2 for almost all values of *q*, indicating that the "classical" ROMA solution, s, is around this value. On the right we see the *normr_ideal* parameter (on the x axis) as a function of s (on the y axis). The minimum value of normr_ideal is automatically selected as the AROMA solution for this range ΔY.

The AROMA spectrum, presented in Figure 11, shows the ROMA solutions s as a function of Y for five different ranges ΔY. The plot on the right hand side shows the rescaled absolute values of the PDFs using the scaling indices corresponding to each range of Y. If the

multifractal spectrum on the left is correct, then we should see a perfect rescaling of all PDFs in that range of scales.

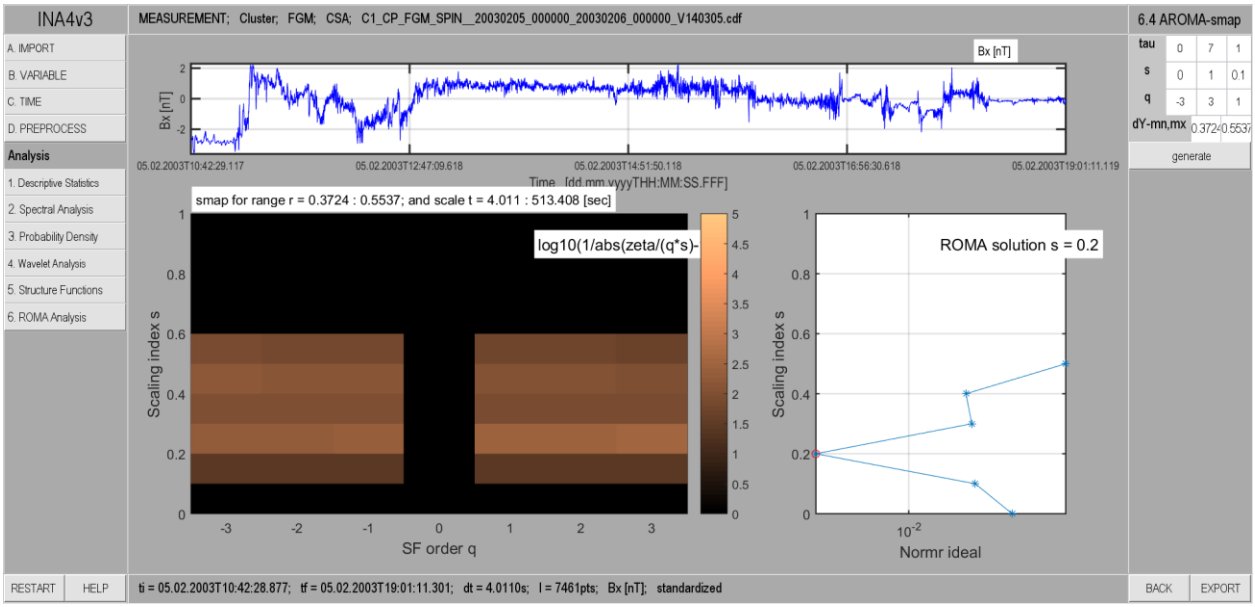

Figure 10. Illustration of the AROMA smap window. The left diagram shows in color coded the goodness of the monofractal scaling quantified as the logarithm of: $F = \frac{1}{ABS(\left(\frac{\zeta}{qs}\right)-s)}$*; brighter colors correspond to better fits. The right plot shows the value of normr_ideal as a function of s – the minimum value corresponds to the solution chosen by AROMA. The user can modify from the left hand side panel the range of scales tau, the range of values s and the discretization, and also the range of moment orders q.*

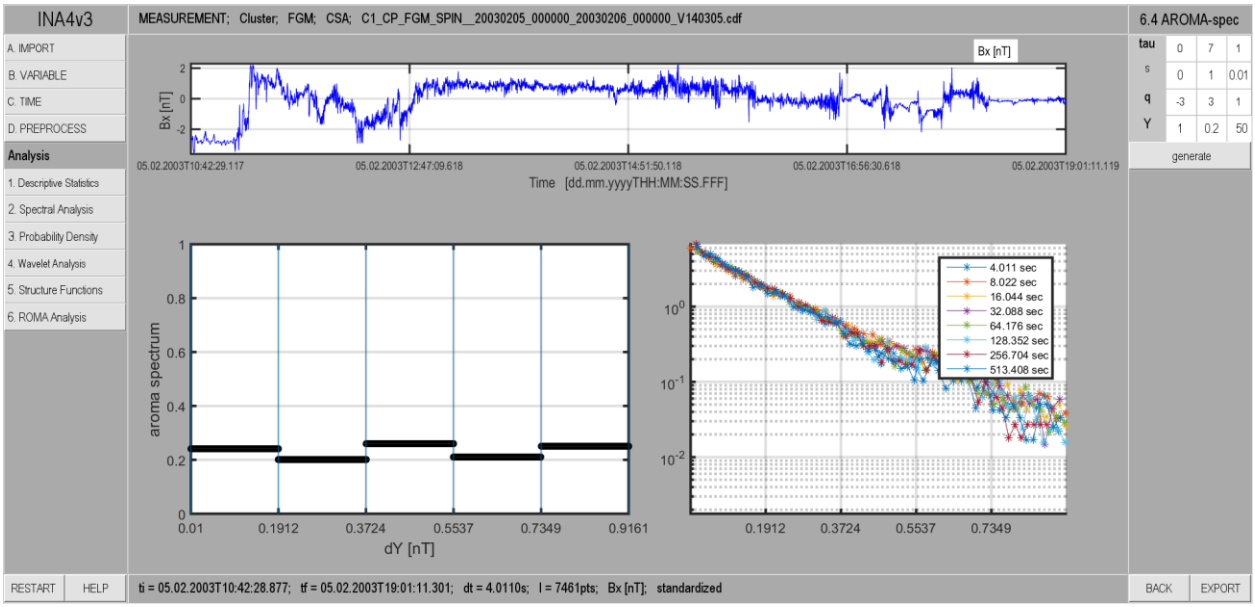

Figure 11. Illustration of the AROMA - spectrum. The plot on the left shows the ROMA spectrum for all bins ΔY computed with the procedure described in the text and illustrated by Figure 10. The right plot shows the Probability Distribution Functions for the rescaled/rank ordered variable Y and the considered range of scales, rescaled with the ROMA spectrum showed at left.